MODELLING FATIGUE SPECTRUM OF AIRCRAFT STRUCTURE UNDER GUST LOADS

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Abstract. The main purpose of the present work is to develop a new computation process in order to perform Fatigue and Damage Tolerance analysis on aircraft structure under gust loads. Random gusts with various amplitudes and directions are considered, which leads to random multiaxial loads on the aircraft structure. In order to analyse these loads and induced high-cycle fatigue, the authors propose an innovative methodology. The latter aims at improving the computation of the dynamic response of the aircraft structure, the stress computation and the fatigue spectrum. From the input Power Spectral Density function, temporal stress sequences have to be obtained in order to be included into the Fatigue Spectrum and combined with other events. The new method proposed by the authors enables to generate correlated stress time histories from the PSD functions. Correlation enables to take into account each component of the multiaxial stress tensor for fatigue analysis. Various turbulence intensities are considered following the statistical law given by the certification. This method provides relevant mathematical results and enables to create directly stress temporal sequences met by the aircraft during its life. These temporal sequences are further used in the overall Fatigue Spectrum of the aircraft. Results of the fatigue analysis will be presented in order to show that the new proposed method improves the turbulence modelling in the Fatigue Spectrum.

1 INTRODUCTION

In the aeronautical context, fatigue of aircraft structures is of crucial interest for aircraft design optimization. Indeed, fatigue analysis aims at ensuring aircraft structural safety, which in particular requires compliance to certification rules [1].

Atmospheric turbulence is an important part of the fatigue induced on the aircraft structure. Gusts with various amplitudes and directions must be considered. The Continuous Turbulence model with Von Karman Power Spectral Density (PSD) has been chosen since it is recommended by certification authorities. In this model, the turbulence is regarded as an ergodic and stationary stochastic process.

The turbulence generates dynamic stresses on the aircraft. The history of these stresses is fundamental for the fatigue and damage tolerance analysis. In order to obtain time histories of stresses, a straightforward approach would consist of generating time signals of stresses from simulated time signals of the turbulence. This approach is always avoided because of its prohibitive computation cost. Therefore, Power Spectral Densities (PSD) will be in force.

Recent results [2] enable the generation of correlated time histories from their Power Spectral Density function. These results are particularly interesting for combining the various components of the stress tensor. The correlated stress time histories will constitute an accurate input for Fatigue and Damage Tolerance analysis.

Several studies have been made in order to determine the gust occurrence probabilities. The patchy and non-Gaussian character of the atmospheric turbulence is now well recognized. Press and Steiner [3] established a formula giving the frequency of exceedance of gust velocities and induced loads and stresses on the aircraft structure. This formula also appears in the certification rules [1]. The generated time histories of stresses induced by gusts may match these statistics of exceedance. Unfortunately, this formula does not take into account the stress correlation. The aim of the presented methodology is to refine Press and Steiner’s result by accounting for the stress correlations.

The paper is organized as follows. In section 2, the continuous turbulence model is presented. In section 3, several useful mathematical results are reviewed in order to provide the mathematical background necessary to understand the new methodology presented in section 4. Finally, in section 5, an application to fatigue computation on an aircraft structure is provided in order to illustrate the benefits of the presented method.

2 CONTINUOUS TURBULENCE MODEL

Continuous Turbulence model is used to represent the atmospheric turbulence as a stochastic process having the Von Karman Power Spectral Density function [4], [5]. Certification authorities recommend choosing this model to perform fatigue analysis of aircraft structure under gust loads. In the Continuous Turbulence model, the wind velocity is regarded as a stochastic stationary ergodic and Gaussian process. The Von Karman Power Spectral Density, represented in Figure 1 is defined by

$$\Phi_{VK}(\omega) = \sigma_{VK}^2 \frac{L}{V} \frac{1+8/3(1.339L/V\omega)^2}{(1+(1.339L/V\omega)^2)^{11/6}} \tag{1}$$

where ω is the angular speed, σ_{VK} is the root mean square of gust velocity, L is the scale of turbulence and V is the true aircraft speed.

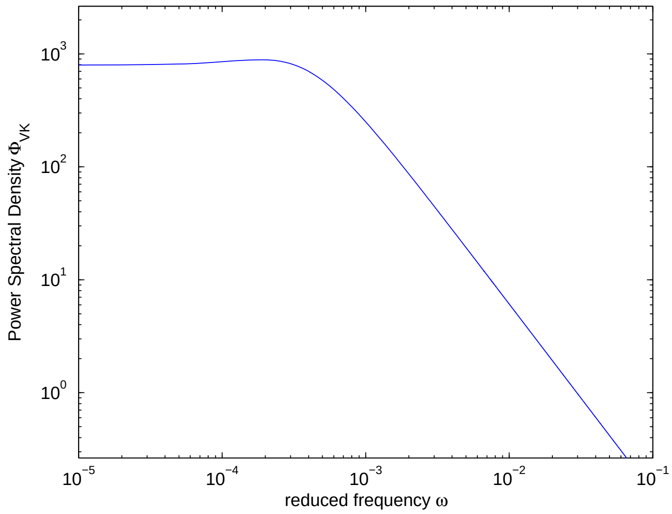

Figure 1: Von Karman Power Spectral Density with σ_{VK}=1 and L=2500ft.

In order to have a representation covering all the turbulence conditions, L is usually fixed to 2500ft (a justification of this value can be found in [6]) and σ_{VK} is considered as a random variable. Press and Steiner [3] established that its probability density function could be expressed as

$$f(\sigma_{VK}) = \sqrt{\frac{2}{\pi}} \frac{P_1}{b_1} \exp\left(-\frac{\sigma_{VK}^2}{2b_1^2}\right) + \sqrt{\frac{2}{\pi}} \frac{P_2}{b_2} \exp\left(-\frac{\sigma_{VK}^2}{2b_2^2}\right). \tag{2}$$

Initially, P_1, P_2 represented respectively the proportion of flight time (or distance) in non-storm and storm turbulence and b_1, b_2 were the scale-parameters values for the individual probability distributions of σ_{VK} for the two types of turbulence. It can also be considered as an empirical equation covering all types of turbulence collectively. P_1, P_2, b_1 and b_2 only depend on altitude.

In this model, the turbulence is assumed to be stationary and Gaussian. Neither of these two assumptions have any physical justification but they enable the use of powerful mathematical results. Nevertheless, gust statistics are quite far from the Gaussian law and then have to be corrected.

3 MATHEMATICAL REVIEW

3.1 Power Spectral Density functions

In the Continuous Turbulence model, the input is a PSD function from which the PSD of loads and stresses can be computed *via* transfer functions. These transfer functions depend on the quantity under consideration, the calculation point, the weight, the stiffness and the configuration of the aircraft. They are computed *via* a Global Finite Element Model and the equations of motion.

The Power Spectral Density of the random signal *x(t)* is a positive real function which describes how the power is distributed along the frequency axis. It is often defined *via* the Wiener-Khintchine theorem as the Fourier transform of the autocorrelation function *R(τ)*:

$$\Phi_x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R(\tau)e^{-j\omega\tau}d\tau$$

with

$$R(\tau) = E[x(t)x(t+\tau)].$$

Here, the signal *x(t)* is assumed to be real, stationary and ergodic. In the present study, atmospheric turbulence as well as stresses are defined by Power Spectral Density functions. Then, it is always possible to generate a stationary and ergodic process that meets this Power Spectral Density function [7].

Without loss of generality, the mean *E[x(t)]* can be assumed to be zero. And by applying the inverse Fourier transform:

$$R(\tau) = \int_{-\infty}^{+\infty} \Phi(\omega)e^{j\omega\tau}d\omega.$$

A noteworthy result [8] is that the standard deviation or root-mean-square (RMS) of the process *x(t)* can be expressed as the square root of the integral of the PSD:

$$\sigma_x = \left(\int_{-\infty}^{+\infty}\Phi_x(\omega)d\omega\right)^{1/2}.$$

Stationarity implies that the signal has an infinite energy and that its Fourier transform does not exist. The Power Spectral Density function can still be determined using the truncated Fourier Transform $\widehat{X_T}(\omega)$ from the following formulas (see [9]):

$$\widehat{X_T}(\omega) = \int_{-T}^{+T} x(t)e^{-j\omega t}dt$$

And

$$\Phi_x(\omega) = \lim_{T\to\infty}\frac{1}{T}\left|\widehat{X_T}(\omega)\right|^2. \tag{3}$$

The above equation involves ergodic hypothesis and assumes that $\tau R(\tau)$ is absolutely integrable (see [8]). Notice that the information on the phase of *x(t)* is not carried by the Power Spectral Density as it can be seen in the above definition (Equation (3)). Then, it is possible to obtain an infinite number of different functions having the same PSD function.

PSD functions also provide very useful results for linear systems. Indeed, the input-output equation can be written as simply as:

$$\Phi_Y(\omega) = |H(\omega)|^2\Phi_X(\omega) \tag{4}$$

with *H(ω)* the transfer function between the excitation source *X(t)* and the output *Y(t)*. This relation is much simpler than the convolution used in the time domain.

3.2 Generation of time signals from PSDs

Shinozuka [7] gives the following formula (using Rice's results [10]) to generate a stationary and zero-mean process from its PSD:

$$x(t) = \mathrm{Re}\left(\sum_{k=0}^{M-1} 2\sqrt{\Phi_x(k\partial\omega)\partial\omega}\exp(i\varphi_k)\exp(2i\pi kt/M)\right)$$

where the φ_k are independent random phases uniformly distributed on $[0;2\pi]$, $\omega_k = k\partial\omega$ and $\partial\omega$ is the frequency sample step. The central limit theorem guarantees that independent and uniformly distributed phases give a Gaussian process.

This result was extended to several correlated time signals in [2]. In particular, it is possible to generate the correlated time signals of the various components of the 2D stress tensor.

$$
\begin{aligned}
\sigma_x(t) &= \mathrm{Re}\left[2\sum_{k=0}^{M-1}\sqrt{\Phi_{\sigma_x}(\omega)\Delta\omega}\,\exp\big(i(\varphi_k + \arg(H_{\sigma_x}(\omega)))\big)\exp(2i\pi kt/M)\right]\\
\sigma_y(t) &= \mathrm{Re}\left[2\sum_{k=0}^{M-1}\sqrt{\Phi_{\sigma_y}(\omega)\Delta\omega}\,\exp\big(i(\varphi_k + \arg(H_{\sigma_y}(\omega)))\big)\exp(2i\pi kt/M)\right]\\
\tau_{xy}(t) &= \mathrm{Re}\left[2\sum_{k=0}^{M-1}\sqrt{\Phi_{\tau_{xy}}(\omega)\Delta\omega}\,\exp\big(i(\varphi_k + \arg(H_{\tau_{xy}}(\omega)))\big)\exp(2i\pi kt/M)\right]
\end{aligned}
\tag{5}
$$

This result is particularly interesting for the fatigue stress life analysis.

3.3 Rice's results used to obtain the Press-Steiner formula

In guts load application, gust or stress statistics are defined *via* exceedance statistics. This enables to count the peaks in the time sequence in compliance with the cycle counting method used for fatigue. A peak is usually defined as the highest value between successive zero up-crossings.
The certification recommends using the Press-Steiner formula (Equation (9)) in order to determine the exceedance frequency of the selected load or stress quantities. This formula was established in 1958 in [3] and officially published by the FAA in 1966 [11]. It is based on Rice's equation which gives the expected number of up-crossings of the threshold value y_s per unit time for a zero-mean stationary Gaussian process $y(t)$:

$$
N(y_s) = N_0 \exp\left(\frac{-y_s^{\,2}}{2\sigma_y^2}\right). \tag{6}
$$

Here, σ_y is the root-mean-square value of $y(t)$ and N_0 is the expected number of zero up-crossings per unit time:

$$
N_0 = \frac{1}{2\pi}\sqrt{\frac{\int_0^\infty \omega^2 \Phi_y(\omega)d\omega}{\int_0^\infty \Phi_y(\omega)d\omega}}.
$$

According to the previously given definition of peaks, we consider that $N(y_s)$ matches the number of peaks per second above a threshold value y_s. In our application, we have to consider that the aircraft structure is exposed to various Gaussian gusts with various times on the overall operational history. Then, the number of peaks of the studied quantity y above y_s can be expressed as

$$
N(y_s) = N_0 \int_0^\infty f(\sigma_y)\exp\left(\frac{-y_s^{\,2}}{2\sigma_y^2}\right)d\sigma_y.
$$

Introducing the gust response factor $\overline{A_y}$ defined by $\overline{A_y} = \frac{\sigma_y}{\sigma_{\mathrm{VK}}}$ in the above equation, we obtain

$$
N(y_s) = N_0 \int_0^\infty f(\sigma_{VK})\exp\left(\frac{-y_s^{\,2}}{2\overline{A}^2\sigma_{VK}^2}\right)d\sigma_{VK}, \tag{7}
$$

where $f(\sigma_{VK})$ is the probability density function of the gust velocity defined by Equation (2). By introducing Equation (2) into Equation (7), we obtain:

$$N(y_s) = N_0 \int_0^{\infty} \frac{P_1}{b_1}\sqrt{\frac{2}{\pi}} \exp\left(\frac{\sigma_{VK}^2}{2b_1^2}\right) \exp\left(\frac{-y_s^{\;2}}{2\bar{A}^2 \sigma_{VK}^2}\right) + \frac{P_2}{b_2}\sqrt{\frac{2}{\pi}} \exp\left(\frac{\sigma_{VK}^2}{2b_2^2}\right) \exp\left(\frac{-y_s^{\;2}}{2\bar{A}^2 \sigma_{VK}^2}\right) d\sigma_{VK}. \tag{8}$$

In order to compute this sum of two integrals, we use a result that can be found in [12]:

$$\int_0^{\infty} \exp\left(-x^2 - \frac{a^2}{x^2}\right) dx = \frac{\sqrt{\pi}}{2} e^{-2a}$$

with $x = \frac{\sigma_{VK}}{\sqrt{2b_1}}$ and $a = \frac{y}{2\bar{A}_y b_1}$.

Then, the first term in Equation (8) becomes

$$N_0 \frac{P_1}{b_1}\sqrt{\frac{2}{\pi}} \sqrt{2} b_1 \frac{\sqrt{\pi}}{2} exp\left(-\frac{2y_s}{2\overline{A_y} b_1}\right) = N_0 P_1 e^{-\frac{y_s}{\overline{A_y} b_1}}$$

Similarly, the second term is $N_0 P_2 e^{-\frac{y_s}{\overline{A_y} b_2}}$. Therefore, we obtain the Press-Steiner formula:

$$N(y) = N_0 \left(P_1 e^{-\frac{y}{b_1 \bar{A}_y}} + P_2 e^{-\frac{y}{b_2 \bar{A}_y}} \right) \tag{9}$$

This formula gives the exceedance frequency of any quantity *y* (for instance acceleration at the centre of gravity, wing bending moment, longitudinal stress...) resulting from the continuous turbulence excitation.

4 FROM CONTINUOUS TURBULENCE MODEL TO FATIGUE STRESSES

4.1 Fatigue analysis in the aeronautical context

In the aeronautical context, the fatigue phenomenon has to be considered on the whole aircraft structure. A Damage Tolerance approach for aircraft structure design and maintenance has been recommended by the certification authorities since 1978. It aims at ensuring that any damage will be detected before it would be critical for the structure integrity. In this approach, stress life analysis is used to compute fatigue damage and crack propagation. Fatigue spectra are used to represent all impacting events regarding fatigue during the life of the aircraft structure. Each of these events generates multiaxial stresses which are obviously interrelated. Therefore, correlated stress time histories are then necessary to perform accurate Fatigue and Damage Tolerance computations.

4.2 Fatigue spectrum

All impacting events relevant for fatigue during the total life of the aircraft structure must be accounted for. Gusts are only one part of these events and it is the only one represented in the frequency domain. In order to build fatigue spectra, various design flight profiles are defined. For each segment of each profile, both random (*e.g.* turbulence) and deterministic (*e.g.* manoeuvres) loads are computed. From these loads, random stress sequences are obtained *via* a finite element model. The complete stress sequence is then used as an input for the Fatigue and Damage Tolerance analysis.
Accounting for the gusts in the fatigue spectrum is a complex issue because of its spectral representation. Building the stress sequence and combining it with other loads requires coming back in the time domain.

4.3 Description of the methodology

From the Von Karman PSD function (1), we can compute *via* the finite element model and transfer functions, the PSD functions of the various components of the 2D stress tensor using Equation (4). This computation is very efficient since it is performed in the frequency domain. From the 3 PSD functions $\Phi_{\sigma x}(\omega)$, $\Phi_{\sigma y}(\omega)$, $\Phi_{\tau xy}(\omega)$, we are able to generate the correlated time signals $\sigma_x(t)$, $\sigma_y(t)$, $\tau_{xy}(t)$ with Equations (5). Introducing Equation (4) in each of these equations, we have

$$\sigma_x(t) = 2\sigma_{VK} Re\left(\sum_{k=0}^{M-1} \sqrt{\left|H_{\sigma_x}(\omega)\right|^2 \Phi_{VK}^N(\omega) d\omega}\, e^{-i\left(\varphi_k + \arg\left(H_{\sigma_x}(\omega)\right)\right)} e^{-\frac{2i\pi kt}{M}}\right) = \sigma_{VK} \sigma_x^N(t)$$

$$\sigma_y(t) = 2\sigma_{VK} Re\left(\sum_{k=0}^{M-1}\sqrt{\left|H_{\sigma_y}(\omega)\right|^2 \Phi_{VK}^N(\omega)d\omega}\, e^{-i(\varphi_k+\arg(H_{\sigma_y}(\omega))}e^{-2i\pi kt/M}\right) = \sigma_{VK}\sigma_y^N(t).$$

$$\tau_{xy}(t) = 2\sigma_{VK} Re\left(\sum_{k=0}^{M-1}\sqrt{\left|H_{\tau_{xy}}(\omega)\right|^2 \Phi_{VK}^N(\omega)d\omega}\, e^{-i(\varphi_k+\arg(H_{\tau_{xy}}(\omega))}e^{-2i\pi kt/M}\right) = \sigma_{VK}\tau_{xy}^N(t).$$

The superscript N indicates that the Von Karman PSD function is normalized in order to highlight the coefficient σ_{VK} in front of the equation. By construction, these time signals follow a Gaussian law. Nevertheless, we can redistribute these time signals according to relevant statistics given by the probability density function $f(\sigma_{VK})$. Indeed, we can define various turbulence intensities σ_{VK}^i having the associated probabilities p_i computed from Equation (2). Thus, we obtain various stress signals $\sigma_x^i(t), \sigma_y^i(t), \tau_{xy}^i(t)$. We can distribute these various stress signals over the fatigue spectrum by computing the flight time ratio spent in turbulence T_{turb}^j for all the flight segments j in the fatigue spectrum. For a given segment j, we have:

$$T_{turb}^j = p_i^j\, T^j\, N^j \tag{10}$$

where N^j is the number of flights containing the segment j with length T^j in the fatigue spectrum. Then, we can distribute the total flight time when the aircraft encounters turbulence in the various flights of the fatigue spectrum. Each flight segment is assumed to be equal in length and to encounter only one level of turbulence intensity. The number of flight segments N_i^j with turbulence intensity σ_{VK}^i is deduced from Equation (10). All the segments N^j are finally randomly distributed in the fatigue spectrum. We know from the results of Section 3.3 that the number of up-crossings of each stress time history is given by Rice's formula (6). Then, the expected number of up-crossings for the whole stress history is equal to:

$$N(y_s) = N_0 \sum_i p_i \exp\left(\frac{-y_s^{\,2}}{2\sigma_y^{i\,2}}\right)$$

which should tend to

$$N(y_s) = N_0 \int_0^{\infty} f(\sigma_{VK}) \exp\left(\frac{-y_s^{\,2}}{2\bar{A}^2\sigma_{VK}^2}\right) d\sigma_{VK}$$

if the various intensities σ_{VK}^i are distributed well enough, *i.e.* if $p_i = f(\sigma_{VK}^i)d\sigma_{VK}^i$. The above equation is the same as Equation (7) which finally leads to the Press-Steiner formula (9). Comparisons of the exceedance statistics are plotted in Figure 2. In this example, we chose 8 levels of turbulence intensity from $\sigma_{VK}^1 = 1$ft/s to $\sigma_{VK}^8 = 8$ft/s which is sufficiently accurate. We note that a single stress time signal matches quite well Rice's result (left figure). On the right-hand side, we compare the Press-Steiner formula with the number of exceedances counted from the whole stress sequence. For low levels of stress, results match perfectly but for high levels of stress, the simulation seems to be damped and the number of exceedances is below the expected results. This discrepancy can be explained by the fact that these events are rare and may not appear in a time simulation of finite length.

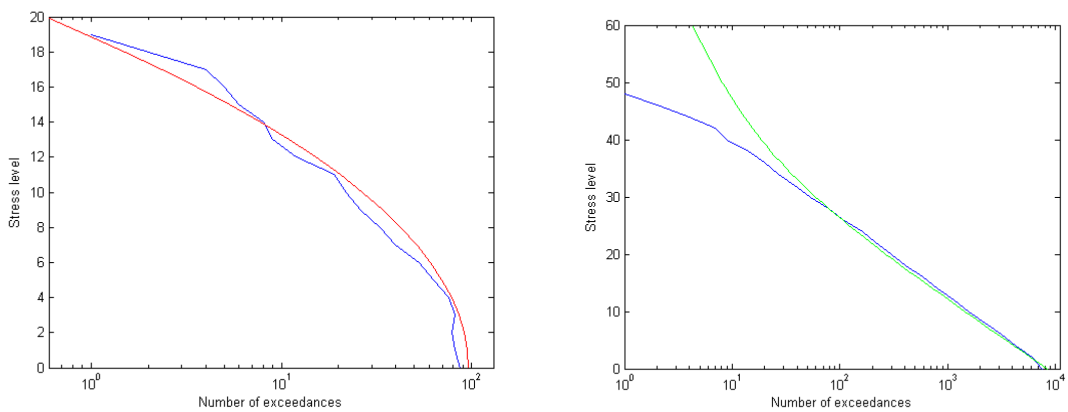

Figure 2: Comparison of exceedances statistics between the analytical formulas from Rice (in red) on the left-hand side, Press and Steiner (in green) on the right-hand side and the generated time signals (in blue).

In our application, these rare events never occur, therefore this discrepancy have not to be taken into account.

This new methodology enables the accurate modelling of gust loads into the fatigue spectrum. The mathematical results about the generation of correlated Gaussian time signals associated to the Press-Steiner formula permit to obtain a long time history of stress having the expected exceedance statistics. Once distributed into the fatigue spectrum, we obtain a very good input to perform the fatigue analysis.

5 FATIGUE AND DAMAGE TOLERANCE APPLICATION

In this section, we provide some results of the fatigue analysis performed from the generated stress sequence input. We will first compute the fatigue damage from which we deduce the number of flight cycles before the crack initiation and then we will perform a crack propagation study which will give the number of flight cycles before the complete failure of the structural component under study. This analysis is realized on a metallic panel of the lower cover of the wing, very sensitive to vertical gust loads. The damage *D* is a dimensionless quantity used to assess the fatigue life of a structural component. By convention, *D=0* if the material is healthy (with no damage at all) and *D=1* if the material is ruined. On the first study, we perform a uniaxial fatigue analysis on a very simple fatigue spectrum which represents the vertical turbulence encountered during one cruise segment with a null steady stress. This very simplistic example aims at illustrating the effect of vertical turbulence increments on a flight segment.
The fatigue damage obtained for this example is equal to $D=3.78\ .10^{-4}$ giving more than 6 million flight cycle fatigue life.
We can develop this example with a fatigue analysis (still uniaxial) on a more consistent spectrum. First, we add stress levels corresponding to ground phases at the beginning and at the end of each flight. Thus, the ground-air-ground cycle is represented, which is of crucial interest for fatigue and crack propagation analysis. We also add a second flight segment: a climbing phase around 26000ft high. At this altitude, the aircraft encounters more severe turbulence patches. The fatigue damage obtained in this case is D=0.0234. This is equivalent to a 84,082 flight cycle fatigue life.
Finally, in order to perform a complete fatigue and damage tolerance analysis, we propose to run a fatigue analysis and a crack propagation study which will consider the various components of the 2D stress tensor. The chosen approach consists in performing the analysis in the most damaging direction for the whole stress history. The fatigue damage *D* depends directly on the stress history sequence. In aeronautics, the stress sequence

$$S(t,\alpha) = \sigma_x(t)\cos^2\alpha + \sigma_y(t)\sin^2\alpha + \tau_{xy}(t)\sin 2\alpha$$

is used, where $t \in [0,T]$ (and *T* is the length of the observed history) and the selected α is the stress direction which maximizes the damage over the observed time sequence. The angle which maximizes the damage corresponding to the spectrum under study is $\alpha=6°$ for fatigue and $\alpha=3°$ for crack propagation. The fatigue damage obtained is D=0.0243 leading to 80,957 flight cycles. This result is quite close to the uniaxial analysis because the chosen component is mostly sensitive to the σ_x component. This is also illustrated by the maximum damaging angle close to 0.
The crack propagation study is performed on the same structural panel and the same spectrum. This calculation is based on a peak-to-peak cycle counting method [13] and on the Elber crack closure concept [14]. This is a retardation model based on the fact that only one part of the stress cycle, larger than the crack opening stress, contributes to the crack opening. The crack opening stress level and the crack growth increment depend on the previous load history and they have to be calculated cycle by cycle. With this model, the number of flight cycles during which the panel will resist to the limit loads can be computed. In this study, the panel is considered as an infinite plate with an initial crack length due to some manufacturing defect. For this typical manufacturing defect ("rogue" flaw concept), the crack length becomes critical, *i.e.* the structure does not withstand the limit loads, after 44,566 flight cycles. All these data represent a first basis in order to establish the inspection program of the aircraft structure.

6 CONCLUSION

In this paper, an efficient computational method for modelling fatigue spectrum of aircraft structure under gust loads has been presented. This approach uses both mathematical results enabling the generation of correlated stress sequences, and atmospheric turbulence modelling with the Press-Steiner formula. This new methodology is efficient in terms of computation cost and provides good results in terms of aircraft dynamic response, stress computation and gust statistics.
Results in terms of fatigue damage and crack propagation are presented in order to illustrate this method. These results are presented on a simplified spectrum. Nevertheless, this example is representative of the aircraft fatigue life and could be straightforwardly extended to a complete spectrum in order to obtain all the necessary data to establish the structure inspection program in compliance with the damage tolerance requirements.

Future work will focus on coupling events. Accounting for the coupling of events may have an impact on the stress sequence. For example, the combination of lateral and vertical gusts as well as the coupling of gusts with manoeuvres should be considered. The Round-the-Clock criterion is currently used to combine vertical and lateral gusts but various methods could be envisaged. The computation of stress sequences from a combined PSD function of oblique turbulence is a first idea which should be developed. We can also imagine the combination of the two stress temporal sequences coming from lateral and vertical turbulence according to the Round-the-Clock criterion.

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